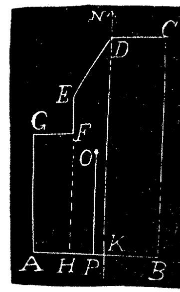
24. Proposed by D. H. DAVISON, C. E., Minonk, Illinois.

For the purpose of locating the most eligible point for a county-seat, it is required to determine the centre of a county whose dimensions are as follows: Beginning at the S. W. corner, thence E. 15 miles, thence N. $33\frac{2}{3}$ miles, thence W. 6 miles to the north end of the meridian running south through the county, thence southwesterly to a point 6 miles W. from the meridian and $9\frac{2}{3}$ miles S. of the north end of said meridian, thence S 3 miles, thence W. 3 miles, and thence S. 21 miles to the place of beginning.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science in Texarkana College, Texarkana, Arkansas; and F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Let $\bar{x}$, $\bar{y}$, be the co-ordinates of the centroid, and divide the county into three parts as in the figure, then we eaily get with A as origin



$$\bar{x}=\frac{\int_0^3\int_0^{21}xdxdy+\int_3^9\int_0^{19\frac{1}{6}+\frac{29}{18}x}xdxdy+\int_9^{15}\int_0^{33\frac{2}{3}}xdxdy}{\int_0^3\int_0^{21}dxdy+\int_3^9\int_0^{19\frac{1}{6}+\frac{29}{18}x}dxdy+\int_9^{15}\int_0^{33\frac{2}{3}}dxdy}$$

$$=8\tfrac{163}{876}\text{ miles.}$$

$$\bar{y}=\frac{\int_0^3\int_0^{21}ydxdy+\int_3^9\int_0^{19\frac{1}{6}+\frac{29}{18}x}ydxdy+\int_9^{15}\int_0^{33\frac{2}{3}}ydxdy}{\int_0^3\int_0^{21}dxdy+\int_3^9\int_0^{19\frac{1}{6}+\frac{29}{18}x}dxdy+\int_9^{15}\int_0^{33\frac{2}{3}}dxdy}$$

$$=15\tfrac{167}{784}\text{ miles.}$$

$\therefore$ Measure east from beginning $8\frac{163}{876}$ miles, then north $15\frac{167}{784}$ miles.

[The proposition that, "The point of the area of a triangle, which has the sum of its distances to all other points of the area a *minimum*, is the centre of gravity of the area," which I think holds for other figures, practically solves problem 24, No. 2. I have made out the proof for the triangle but it occupies two pages. R. J. ADCOCK, Larchland, Illinois.]

Also solved by *P. S. BERG*.

A CORRECTION.—On page 246 of the MONTHLY, my remarks in the lower four lines *above* the *Note*, are not true and should be expunged. They were hastily made upon insufficient investigation. In prob. 20, those remarks hold almost true, but in the general problem they can not ever be true. My solution is not at all affected by those misstatements. The solution may be more easily understood by adding, that, "when Sirius rises, some point of the ecliptic is then rising, and as the Sun is always on the ecliptic the Sun must be at that point, in order to rise synchronously with Sirius.

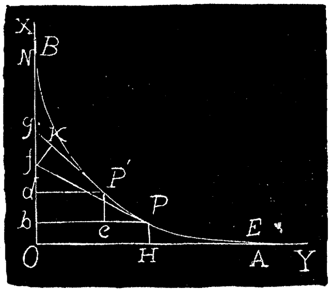
S. H. WRIGHT.

34. Proposed by GEORGE LILLEY, Ph. D., LL. D., Park School, Portland, Oregon.

A hare is at O, and a hound at E, 40 rods east of O. They start at the same instant each running with uniform velocity. The hare runs north, The hound runs directly towards the hare and overtakes it at N, 320 rods from O. How far did the hound run?

III. Solution by J. M. BANDY, A. M., Professor of Mathematics in Trinity High School Trinity, North Carolina.

Let $OE=a$, $ON=b$, $r=$ rate of hound, $r'=$ rate of hare, $\frac{r}{r'}=n$, ratio of their speed; co-ordinates of $P=(x,y)$, and $z=EP$. By the theory of curves, $dz=\sqrt{1+\frac{dx^2}{dy^2}}\,dy\ldots.(1)$. Now $bf=\frac{ydx}{-dy}$, (y being a decreasing function of x is negative).



$\therefore\ Of=x+\frac{ydx}{-dy}$. By the problem, $\frac{EP}{x+\frac{ydx}{-dy}}$

$=\frac{r}{r'}=n$, or $EP=n\left(x+\frac{ydx}{-dy}\right)\ldots\ldots(2)$.

$\therefore\ x+\frac{ydx}{-dy}=\frac{1}{n}\int\sqrt{1+\frac{dx^2}{dy^2}}\,dy\ldots\ldots(3)$. Put

$\frac{dx}{dy}=p$, then $dx=pdy$, and we have, $x-py=\frac{1}{n}\int\sqrt{1+p^2}\;dy\ldots.(4)$. Differentiating (4) and observing that $dx=pdy$, we have, $-ydp=\frac{1}{n}\sqrt{1+p^2}\,dy\ldots(5)$.

From (5), $\frac{dp}{\sqrt{1+p^2}}=-\frac{dy}{ny}\ldots.(6)$. Integrating (6), we have, $\log[p+\sqrt{1+p^2}$ $=-\frac{1}{n}\log y+C=\frac{1}{n}\log\frac{1}{y}+C\ldots.(7)$.

When $x=0$, $p=0$, and $y=a$, and $C=\frac{1}{n}\log a$.

$\therefore\ \log[p+\sqrt{1+p^2}=\frac{1}{n}\log\frac{a}{y}\ldots.(8)$. Passing from log, $p+\sqrt{1+p^2}$ $=\left(\frac{a}{y}\right)^{\frac{1}{n}}\ldots(9)$. Solving for p, we have, $2p=\left(\frac{a}{y}\right)^{\frac{1}{n}}-\left(\frac{y}{a}\right)^{\frac{1}{n}}\ldots(10)$. But $p=\frac{dx}{dy}$.

$\therefore\ 2dx=\left(\frac{a}{y}\right)^{\frac{1}{n}}dy-\left(\frac{y}{a}\right)^{\frac{1}{n}}dy\ldots.(11)$. Integrating (11), $2x=\frac{a^{\frac{1}{n}}y^{1-\frac{1}{n}}}{1-\frac{1}{n}}$ $-\frac{y^{1+\frac{1}{n}}}{a^{\frac{1}{n}}\left(1+\frac{1}{n}\right)}+C\ldots.(12)$. When $x=0$, $y=a$, and (12) gives val. of $C=\frac{2na}{n^2-1}$.

$$\therefore\ 2x=\frac{a^{\frac{1}{n}}y^{1-\frac{1}{n}}}{1-\frac{1}{n}}-\frac{y^{1+\frac{1}{n}}}{a^{\frac{1}{n}}\left(1+\frac{1}{n}\right)}+\frac{2na}{n^2-1}\ldots.(13.$$

This is the equation of the curve described by the dog, and it is called the "Curve of pursuit." When the dog overtakes the hare, $y=0$, $x=ON=b$. $\therefore$ (12) becomes, $\frac{na}{n^2-1}=b$....(13). Solving (13) as a quadratic in n, we have, $n=\frac{1}{2b}(a\pm\sqrt{4b^2+a^2})$, or $n=1.0644+$. Substituting this value of n in (2), remembering that when the dog overtakes the hare, $EP=s$, $x=320$, $y=0$, we have, $s=1.0644\times320=340.624+$ rods.

PROBLEMS.

33. Proposed by Professor ALEXANDER ROSS, C. E., Sebastopol, California.

From a point P without a rectangular field ABC, the distances PA, PB, and PC measured to the corners are, respectively, 70, 40, and 60 chains. What is the area of the field?

34. Proposed by THOS. U. TAYLOR, C. E., M. C. E., Department of Engineering, University of Texas, Austin, Texas.

Given a variable parrallelogram $ABCP$, where P remains fixed. A moves on an irregular plane curve (closed) and C moves on another plane curve (closed) whose plane is parallel to the plane of (A) curve. The generator PC moves completely around and returns to its initial position, AB always moving parallel to PC, and, of course, returns to its initial position. If distances between planes (A) and ($C=h$), show by elementary mathematics and without using theorem of Koppe that volume of solid generated by variable parallelogram $ABCP=\frac{1}{2}h$ (area generated by $AP+$ area generated by BC).

QUERIES AND INFORMATION.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SPACE.

Space is an entity, outside of the human mind, extended in three directions at right angles to each other, continues, immaterial, immmovable, inflexible and illimitable. It is an entity, sui generis, neither psychical nor physical.

It is cognized but not created by the mind of man and is, doubtless, what it is cognized to be.

A fourth dimension has never been discovered.

Arthur Willink in "The World of the Unseen," pages 90 and 91, locates his hypothetical "Higher Space" in an unknowu direction from our space.